Team 105

Problem A: Protecting travellers to Mars

Abstract

This paper analyses and evaluates a possible flight plan from Earth to Mars such that the radiation dosage absorbed by the spacecraft passengers is the least by launching the spacecraft between 43.64 to 55.31 deg relative to the magnetic axis of the Earth (to avoid the core of Van Allen Belt) during beginning or end of each solar activity cycle and calculated the fastest possible route to Mars, that is in 259 days.

There is also an attempt to find the best possible and practical radiation shielding method for sending a 1000 cubic metres habitat along with the extra mass of the faraday cage and all the radiation proof materials like lead, plexiglass, and kevlar, to Mars which came out to be around 6.33 x 10^5 kg of extra mass.

Parameters like, shape of habitat, permitted solar radiation levels and different shielding materials, dosage and exposure rate reduction are taken into consideration while designing a shield for the habitat.

CONTENTS

1 **Introduction**

This project has an aim to send humans to Mars and protecting them from extreme radiations in Van Allen belt and the solar activity. This can be possible if the trajectory avoids the most active parts of belt at the same when the activity of the Sun's cycle is minimum. Also, the Galactic cosmic radiations from outside the solar system from every direction also take part in causing harm to humans and the electronics in spacecrafts. For the radiation dosage absorbed to be least, the flight time and path between Earth and Mars should be minimum.

The habitat, required here, should be of 1000 m^3 and shielded from the most radiations. The shape of the vessel would also determine the amount of radiation in contact with surface of habitat.

Also, either the shield should be thick enough or made up of efficient materials, keeping in mind the cost and the extra mass that would be required to send along with habitat vessel. The material chosen should be such that it does not generate secondary radiations or nuclear reactions and should prevent x rays too. Coming in contact with high energy charged particles can also increase the temperature of vessel to high extent hence the material should also be heat resistant.

2 **Notations used**

Symbols	Meaning	Numeric Value
Re	Radius of Earth	6378.1 km
r	Radius of wire	
n	Number of wires	
M	Mass of Earth	$5.976 \cdot 10^{24}$ kg

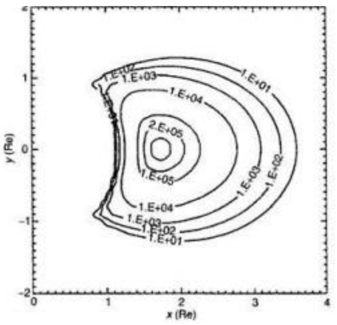
*notations not mentioned are defined alongside

3 **Determination of flight path**

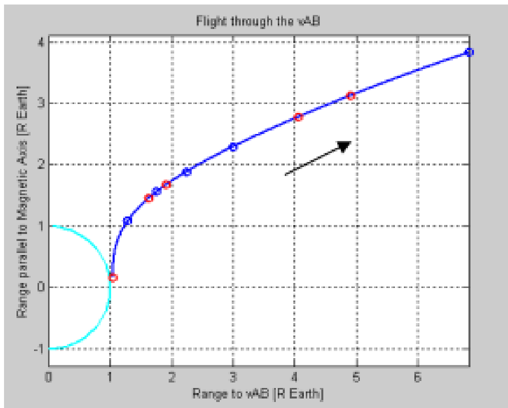
Van Allen belt

The flight path to the Mars crosses the Van Allen radiation belt, a zone with free protons, electrons and other high energy charged particles which is formed as the Earth's magnetic field captures the charged particles from solar winds. So, shielding is required for the sustainability of humans and electronics.

Proton radiation belt



Flight path wrt. Geomagnetic reference system



For the numerical simulation the following differential equation is integrated.

$$\ddot{\underline{r}} = -\frac{\Gamma \cdot M}{|\underline{r}|^{3}} \cdot \underline{r}$$

r is the vector from the centre of the Earth to the space craft; Γ is the gravitational constant ($6.674 \cdot 10^{-11}$ $m^3/(kg \cdot s^2)$); M is the mass of the Earth ($5.976 \cdot 10^{24}$ kg) and $\ddot{r}$ is the 2nd time derivative of r, i.e. the acceleration vector. The reference system is Earth fixed (not rotating with the Earth, i.e. inertial): the origin is in the centre of the Earth, the x-axis in the direction of the vernal equinox, the z-axis = Earth axis in the direction of the North Pole and the y-axis results from the right-handed system.

Suppose initially the trajectory begins parallel to the magnetic axis. For the calculation for the angle

$$\Theta = \arctan \frac{dy}{dx}$$

$1<x<1.05$ and $0.5<y<1$, $\theta_1 = 43.64$ deg

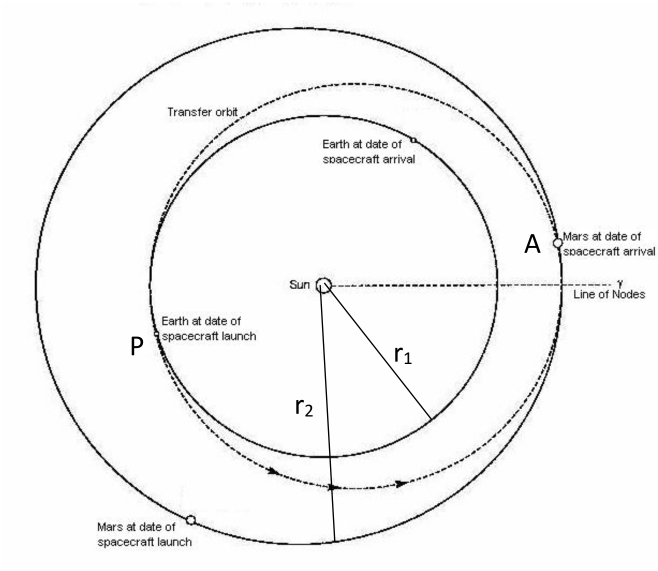
$1.05<x<1.2$ and $1<y<2$, $\theta_2 = 55.31$ deg

Therefore for the trajectory to avoid the central part of the Van Allen and the free proton belt, the trajectory angle should vary from **43.64** to **55.31 deg** relative to the magnetic axis of the Earth.

Flight time and path to Mars

To protect the passengers from radiations, not only radiation shielding is required but also the right timing of launch is necessary and the flight time and path should be as minimum as possible in order to spend least time in contact with Galactic Cosmic radiations.

The typical time during Mars's closest approach to the Earth every 1.6 years is about 260 days. In the nine months it takes to get to Mars, Mars moves a considerable distance around in its orbit, about 3/8 of the way around the Sun. Practically, this means there is only one launch window every 26 months.



Hohmann Transfer Orbit

Kepler's 3rd law as used here. For completeness that form is re-derived below. The law states that for all objects orbiting the Sun

$$T^2 / a^3 = \text{constant}$$

where **T** is the orbital period and **a** the semi-major axis, half the length of the orbital ellipse (**a=r** in circular orbits). The constant is the same for all objects orbiting the Sun, including of course the Earth. Its exact value depends on the units in which **T** and **a** are measured. That value becomes very simple if **T** is measured in years and **a** in AU. Inserting in the 3rd-law equation the values for Earth gives

$$T = 1 \text{ year}$$

$$a = 1 \text{ AU}$$

$$\text{then, } T^2 / a^3 = 1$$

Consequently in these units the constant also equals 1, and that value can be used for any planet. On multiplying both sides of the equation by a^3

$$T^2 = a^3$$

This again holds for any orbit around the Sun, including the one of a spacecraft in a transfer ellipse. The length PA of that ellipse is, in AU,

$$r_1 + r_2 = 1 + 1.523691 = 2.523691 \text{ AU}$$

That is the "major axis" of the orbital ellipse, and half its length equals the semi-major axis a. Therefore

$$a = 1.261845$$

$$a^3 = 2.00918 = T^2$$

$$T = 1.4174 \text{ years}$$

The one-way transit time to Mars is half

= 0.70873 years

= 258.686 days

It takes Mars 1.8822 years for a full orbit of 360^0. Therefore, assuming a circular orbit and uniform motion (a less accurate approximation for Mars than for Earth), in 0.70873 years it should cover

$$360^0 * (0.70886 / 1.88) = 135.555^0$$

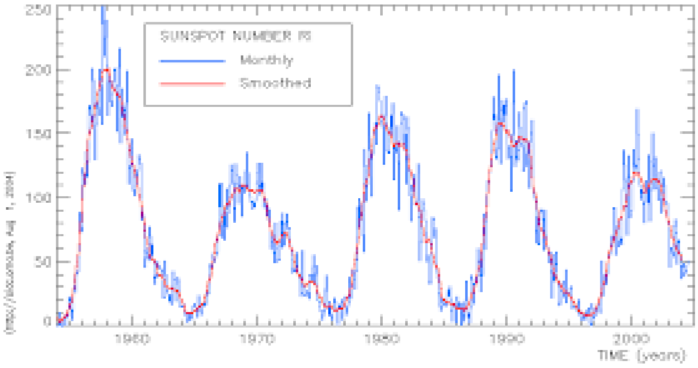
We therefore launch when Mars in its orbit is **135.555^0** away from point A

Activity of Sun

Solar cycle is an approximately 11-year period of varying solar activity including solar maximum where the solar wind is strongest and solar minimum where the solar wind is weakest. Galactic cosmic rays create a continuous radiation dose throughout the Solar System that increases during solar minimum and decreases during solar maximum. The middle of the solar cycle is the solar maximum, or when the Sun has the most sunspots. As the cycle ends, it fades back to the solar minimum and then a new cycle begins.

Solar proton events (SPEs) are bursts of energetic protons accelerated by the Sun. They occur relatively rarely and can produce extremely high radiation levels. Without thick shielding, SPEs are sufficiently strong to cause acute radiation poisoning and death.

Timing of the launch to Mars



Hence, there are two windows in every 11 years when the spacecraft can be launched to Mars. One window is between the beginning of the solar cycle and the middle of the cycle. The other window is between the middle and end of the cycle. However the most suitable time of the launch would be towards the beginning or towards the end of the solar activity cycle when the solar activity is minimum.

Therefore, suitable years for the launch would be **mid-2019** to **early 2021** or **late 2028** and **late 2030** and the journey of 258.686 days to Mars should be accommodated in there 2 windows.

4 Design of habitat vessel

a. Shape of habitat

Shape of the habitat is an important factor in minimizing the surface available for direct contact with radiations and also to have the prescribed volume. Thus habitat vessel should be such that it has the least surface area to volume ratio. Therefore the proposed shape is sphere.

Practicality of Sphere vs Cube habitat

Spherical habitat does not have edges decreasing the probability of radiation leak from corners. However, in cubes/cylinders, weaker joints are present. Moreover, in outer space, there is negligible gravitational acceleration so the passengers can easily move about and round the spherical habitat and proposals like the 'Bernal Sphere' prove the possibility of this habitat. Hence, spherical habitats are practical and possible.

b. Dimensions of the spherical habitat:-

Given volume = **1000 m^3**

So, Inner most radius of the habitat = **6.202 m**

Inner surface area = **483.421 m^2**

c. Radiation reduction around habitat

i. Without shielding

The three basic methods used to reduce the external radiation hazard are time, distance, and shielding. Good radiation protection practices require optimization of these fundamental techniques.

1. Time -
The amount of radiation an individual accumulates will depend on how long the individual stays in the radiation field, because:
Dose (mrem) = Dose Rate (mrem/hr) x Time (hr)
Therefore, to limit a person's dose, one can restrict the time spent in the area. How long a person can stay in an area without exceeding a prescribed limit is called the "stay time:
Stay Time = DoseRate (mrem/hr) Limit (mrem)
2. Distance –
The amount of radiation an individual receives will also depend on how close the person is to the source.

3. The Inverse Square Law -

Point sources of x- and gamma radiation follow the inverse square law, which states that the intensity of the radiation (I) decreases in proportion to the inverse of the distance (d)

$$I \propto \frac{1}{d^2}$$

$$I = \frac{k}{d^2}$$

where k is the constant of proportionality

For different intensities,

$$\frac{I_1}{I_2} = \frac{d_2^2}{d_1^2}$$

Therefore, by knowing the intensity at one distance, one can find the intensity at any other distance.

Gamma Exposure Rate Formula

The exposure rate (E_R) from a gamma point source can be approximated from the following expression:

$$E_R = \frac{6\mathrm{CEf}}{d^2}$$

Where:

C is the activity of the gamma emitter, in millicuries

E is the gamma ray energy in MeV

f is the fraction of disintegrations yielding the gamma of energy E

d is the distance from the source in feet

Galactic Cosmic Radiation

ii. With shielding

When reducing the time or increasing the distance may not be possible, one can choose shielding material to reduce the external radiation hazard. The proper material to use depends on the type of radiation and its energy.

The types of radiations that habitat needs to be shielded from are as follows:-

1. Alpha and Beta Radiation
   Alpha particles are easily shielded. A thin piece of paper or several cm of air is usually sufficient to stop them. Thus, alpha particles present no external radiation hazard. Beta particles are more penetrating than alpha particles. Beta shields are usually made

of aluminum, brass, plastic, or other materials of low atomic number to reduce the production of bremsstrahlung radiation.

2. X and Gamma Radiation

   Both x-rays and gamma rays are forms of high-frequency *ionizing radiation*, which means they have enough energy to remove an electron from (ionize) an atom or molecule. Ionized molecules are unstable and quickly undergo chemical changes.

3. Galactic cosmic radiations

   They are slowly varying, highly energetic background source of energetic particles that constantly bombard Earth. GCR originate outside the solar system and are likely formed by explosive events such as supernova. These highly energetic particles consist of essentially every element ranging from hydrogen, accounting for approximately 89% of the GCR spectrum, to uranium, which is found in trace amounts only. These nuclei are fully ionized, meaning all electrons have been stripped from these atoms.

Monoenergetic x- or gamma rays collimated into a narrow beam are attenuated exponentially through a shield according to the following equation:

$$I = I_o\, e^{-\mu x}$$

Where

x is the thickness of shielding material
I is the intensity outside of a shield of thickness x
I_o is the unshielded intensity
μ is the linear attenuation coefficient of the shielding material and it is the sum of the probabilities of interaction per unit path length by each of the three scattering and absorption processes

It shall now be assumed a mass attenuation coefficient μ_m as linear attenuation coefficients are proportional to the absorber density, which usually does not have a unique value

$$\text{So, } \mu_m = \frac{\mu}{\rho}$$

where ρ = density (g/cm3)

Now, substituting μ ,

$$I = I_o\, e^{-\mu m \rho x}$$

Material of Shield

Before calculating the thickness required to shield the habitat we first need to choose the right material to shield all the mentioned types of radiations as when working with a radionuclide that emits multiple types of radiation such as beta particles and gamma radiation, it is sometimes necessary to shield with several materials.

NOTE:- By shielding the less penetrable radiation type first then proceed to shield the more penetrable type would decrease both scattering and the total amount of shielding material required.

i. Plexiglass

The less penetrating beta radiation can first be shielded with a layer Plexiglass, thereby slowing or stopping the beta particles while reducing the production of bremsstrahlung. Moreover, plexiglass is a hydrogen rich substance which increases efficiency of shielding.

ii. Lead

Lead is a common shielding material for x-rays and gamma radiation because it has a high density, is inexpensive, and is relatively easy to work with.

iii. Kevlar

It is an infusible, wholly aromatic polymer that can strictly be described as nylon T,T - but rarely is. Manufactured only as a fibre (by solution spinning), it has a very high thermal stability and temperature and flame resistance.

Calculating the thickness of individual materials:-

Assume, there is maximum constant radiation exposure rate from a point source outside the habitat that is 9R/hr and the acceptable radiation exposure rate is 0.0005R/hr

So, I_0 = 9R/hr

I = 0.03R/hr

$$I = I_o e^{-\mu x}$$

For Lead,

Density = 11.35 gm/cm^3

$\mu = \mu_m . \rho$

$= (0.114 cm^2/gm)(11.35 gm/cm^3)$
$= 1.63\ cm^{-1}$

x = 3.5 cm

For Plexiglass,

Density = 1.185 gm/cm^3

$\mu = 0.12\ cm^{-1}$

x = 40 cm

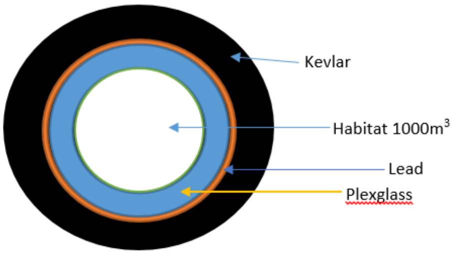
For Kevlar,

Density = 1.44 gm/cm^3

$\mu = 0.902\ cm^{-1}$

$x = 6\ cm$

Habitat 2D view

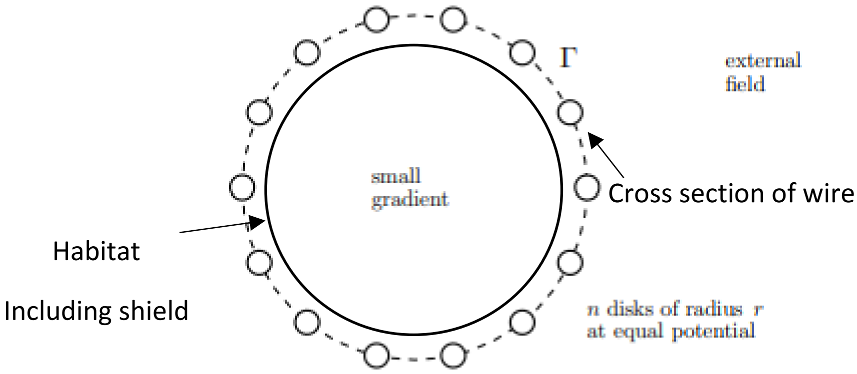


Faraday Cage

Faraday cage effect, whereby a wire mesh or metal screen serves to block electric fields and electromagnetic waves. A Faraday cage operates because an external electrical field causes the electric charges within the cage's conducting material to be distributed so that they cancel the field's effect in the cage's interior. This phenomenon can be used to protect sensitive electronic equipment and make the habitat vessel free from EM radiations.

Faraday cages cannot block stable or slowly varying magnetic fields, such as the Earth's magnetic field (a compass will still work inside). However, the magnetic fields in the outer space are varying and travel in all directions. So, to a large degree, though, they shield the interior from external electromagnetic radiation.

Now, let



Top 2D view of spherical habitat vessel

Given a bounded simply connected open subset of the plane with smooth boundary Γ. Here, real function ϕ(z) that satisfies the Laplace equation,

$$\nabla^2 \phi = 0 \quad (2.1)$$

(Assume Γ is the unit circle)

in the region of the plane exterior to the disks, and the boundary condition,

$$\phi = \phi_0 \quad (2.2)$$

on the disks. Equation (2.2) asserts that the disks are conducting surfaces at equal potential; here $\phi 0$ is an unknown constant to be determined as part of the solution. We emphasize that (2.2) fixes $\phi(z)$ to a constant value on disks of finite radius $r > 0$.

It is also needed to specify some external forcing and appropriate boundary conditions at infinity. Our numerical examples will focus on the case where the external forcing is due to a point charge of strength 2π located at the fixed point $z = z_s$ outside

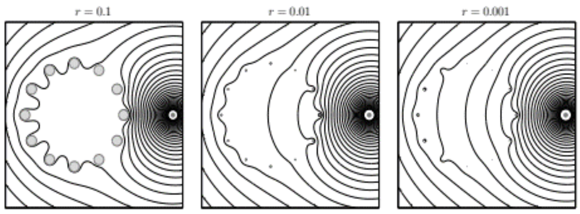
Γ, stipulating that

$$\phi(z) = \log(|z - zs|) + O(1) \text{ as } z \to zs, \quad (2.3)$$

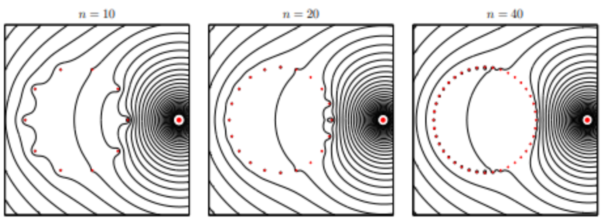
$$\phi(z) = \log(|z|) + o(1) \text{ as } z \to \infty \quad (2.4)$$

Equation (2.4) implies that the total charge on all the disks is zero, though the charge on each individual disk will in general be nonzero.

Now point charge at zs = 2, for fixed n = 12 and varying wire radii r. One sees that the screening effect weakens as $r \to 0$, and in the limit $r = 0$, there will be no screening at all. The dependence on r is logarithmic.

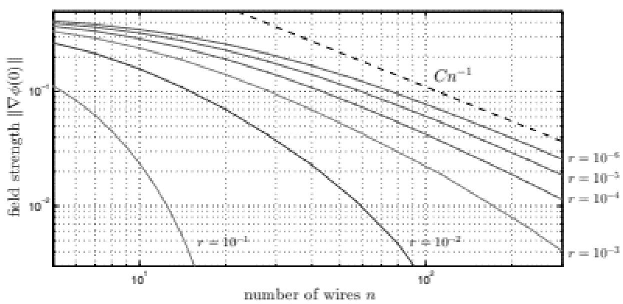


Now fixing r = 0.01 and varies n, showing results for n = 10, 20, 40. With each doubling of n, the field in the cage weakens, but only by approximately a factor of 2.



So, the electric field strength is given by,

$$|\nabla\phi(0)| \approx \frac{-2\log r}{n|z_s|}.$$



Thus the strength of the screening effect is linear in n, logarithmic in r, and linear in |zs|.

The homogenized approximation can be described as follows. Suppose the constant separation between neighbouring wire centres (measured with respect to arc length along Γ) is $\varepsilon = |\Gamma|/n \ll 1$, where $|\Gamma|$ is the total arc length, and the radius of each wire is $r \ll \varepsilon$. The crucial scaling parameter that determines the effectiveness of the screening is

$$\alpha = \frac{2\pi}{\varepsilon \log(\varepsilon/2\pi r)}.$$

If $\varepsilon \gg 1/\log(\varepsilon/r)$, then $\alpha \ll 1$ and the wires are too thin for effective screening. If $\varepsilon \ll 1/\log(\varepsilon/r)$, then $\alpha \gg 1$ and the screening is strong.

However, when n > 40 and $r = 10^{-1}$ m, $|\nabla\phi(0)| \approx 0$, which implies that the electromagnetic shielding is perfect. Hence a continuous conductive shell or overlapping wires is more effective in shielding than a wire mesh.

Now, dimensions for the Faraday cage to be used over the habitat vessel are as:-

Radius of the spherical cage (from centre of habitat) = **6.697 m**

Radius of each wire = **0.1 m**

Number of wires to be used to create almost a conductive shell over the habitat (n)

$$= \frac{2\pi R}{2r} = 210 \text{ (approx.)}$$

Still, let n = **180** (because of the possible expansion in wires' dimensions if temperature increases)

Material of the wires be aluminium (highly conductive and less denser than copper), where density of aluminium = 2,710 kg/m^3

Now, volume of 180 wires = 180 x $\pi r^2 h$ (here $h = \pi R$)

= 118.974 m^3

So, mass of total aluminium used = 2,710 kg/m^3 x 118.974 m^3

= 322419.901 kg

= **3.224 x 10^5 kg** (1)

Total mass of the habitat shield

Volume used = $\frac{4\pi(R_2^2 - R_1^2)}{3}$

Mass = Density/ Volume used

- Mass of Plexiglass used:

R_1 = 6.202 m

R_2 = 6.202+0.4 = 6.602 m

Volume used = 206.085 m^3

Density = 1.185 gm/cm^3

Mass = **244210kg** (2)

- Mass of Lead used:

  R_1 = 6.602 m

  R_2 = 6.602+0.035 = 6.637 m

  Volume used = 19.273 m^3

  Density = 11.35 gm/cm^3

  Mass = **21874kg** (3)

- Mass of Kevlar used:

  R_1 = 6.637 m

  R_2 = 6.637+0.06 = 6.697 m

  Volume used = 33.540 m^3

  Density = 1.44 gm/cm^3

  Mass = **48297kg** (4)

From (1) (2) (3) and (4)

Total mass = 636800.6 kg = **6.36 x 10^5 kg**

5 **Tissue dosage of radiation** during journey to Mars (259 days)

1. Without shield

   = 9R/hr x 24 x 259 x 10mSv = 559.48 Sv

2. With shield
   = 0.03R/hr x 24 x 259 x 10mSv = 1.86 Sv (which is under the limit of dosage per year)

Reduction in dosage annually = (559.48 - 1.86) x 100/ 559.48 = **99.667%**

6 **Conclusion**

The calculated flight path using transfer orbit, trajectory angle and the solar activity minima window are the best ways to avoid most part of the radiations from van allen belt and the sun and also the path is such that the passengers spend minimum time among the radiations so that there is enough time for the temporary DNA and tissue damage to repair quickly. The habitat designed, is the optimum one as the shape used is sphere which provides the least surface area to radiations and the materials used to create the shield are also the best as they are cheap, low density, kelvar being highly heat resistant would provide protection in case of temperature increment.

We have assumed that most radiations are x ray and gamma radiations after crossing the van allen belt and these radiation emerge from a point source and are constant at their maximum value. It is also assumed that the solar electron belt will have negligible effect to the habitat because of the Faraday cage.

However, in reality, there are more types of radiations and presence of asteroids etc in space which might cause damage to the habitat as the outer cage of aluminium is not the most rigid substance but is efficient in shielding from the radiations, mainly from changing electric fields.

Further extension of this work is required to find a possible way to make such habitat and shield which is lighter, more rigid, provides more protection from charged particles and is more compact. For now, the mentioned solutions in this paper are efficient to overcome the challenge to send humans to Mars while protecting them from radiations.

7 **References**

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