Table Tennis Ball Sizes

Team 414 : Problem B

Abstract

In the year 2000, the International Table Tennis Federation changed the official ball diameters from 38mm to 40mm to make it a better spectator sport. The International Table Tennis Federation (ITTF) had done so to decrease the overall speed of the ball and increase the average rally count, so as to turn table tennis into a more fun spectator sport. In this study, the effect of changing ball diameters on television spectator enjoyment is analysed. The effect of increasing ball size on the overall horizontal velocity of the ball will be investigated for each of three kinds of ball strokes; The direct smash, the loop stroke and the chop stroke. These horizontal velocities will be compared with the desired optimal velocity of 11.88m/s, which is based on the visual tracking speed of television spectators. The optimal ball size to yield the most fun for spectators is also determined. For the direct smash, the ball speed decreases with increasing ball diameter as expected, indicating that the 44mm ball was most fun to watch. For the loop stroke ball, the 44mm diameter ball had a velocity closest to that of the optimal velocity. For the chop stroke ball, the analysis was less useful due to the nature of the stroke, but the displacement graph still followed expected results. Overall, the 44mm ball was found to be closest to the optimal size, and the analysis proved accurate for predicting table tennis ball behaviour.

INTRODUCTION

Table tennis has had several changes to the rules and regulations of the sport, introduced in years 2000 and 2001, one of which is the increase in ball diameter from 38mm to 40mm in year 2000. The main reason stated by ITTF to increase the diameter of the ball is to slow the ball down and to increase the rally length so as to make table tennis a more fun spectator sport. Table tennis has long been considered a bad spectator sport due to its high speed of play, and inherent nature of being difficult to track due to the small size of the ball. Increasing the ball thus has a multitude of effect making the sport more spectator friendly: i) a bigger ball is easier to spot; ii) an increased drag force thus slowing the ball down faster; and iii) an increased moment of inertia thus decreasing spin. These effects also allow the players to more effectively hit the ball with less uncertainty from the spin and thus keeping the ball in play longer, with a larger number of rallies per point scored. However, this also simplify the game for the players, who now display a lower level of techniques and control at every level of play of the game. This, conversely, will also lowers the perception of fun within the spectators who gain more utility watching a challenging sport. Thus, we found a balance between the two extremes, with the optimal speed of the ball being 11.88m/s, which is the average limit of the visual acuity of the eye for tracking a moving object, after crossing the net so as to allow the ball to be slow enough to be adequately tracked by the spectators' eyes, yet still sufficiently fast enough to warrant a high level of play from the players. The reason for choosing only to address the television spectators will be discussed in a later section.

To analyse if changing the ball size would make the game more fun to watch on television, we determine the top translational speed of three different kinds of rallied balls: a smash, a top spin loop and a back spin chop. If the average speed of each of these rallies is reduced, this implies that the game becomes easier and more fun to watch. The reduction in average speed can only be so much before the game becomes too slow for entertainment however. Therefore, an optimal speed based on the average tracking speed of television spectators is obtained and used as a benchmark for comparison.

In this study, the motion or trajectory of a table tennis ball is analysed classically with kinematics and aided with numerical analysis using software like Scilab and C++.

THEORY

The initial height and position of the ball for each of the three strokes is the same. The ball is 50cm above the edge of the table and is projected diagonally across the table to cover the maximum distance possible of about 313cm. This distance is taken, assuming that spectators enjoy watching such cross-court shots as they are technically more difficult to execute.

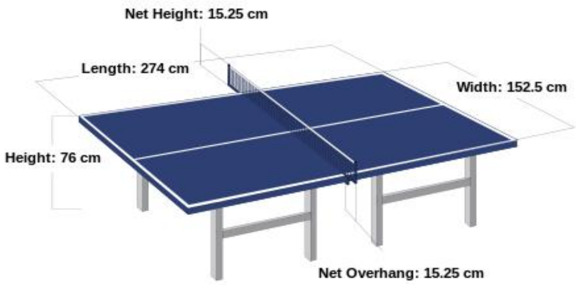


Fig. 1 Dimensions of a Table Tennis Table

The general equation used to map the trajectory and velocity of the ball is found below:

$$m_{ball}\frac{d^2}{dt^2}\vec{r} = m_{ball}\vec{g} + (4\pi C_M {r_{ball}}^3 \rho\vec{\omega} \times \vec{v} - \frac{C_d\rho A}{2}|\vec{v}|\vec{v}) \quad (1)$$

, where ρ is the density of air, $\vec{\omega}$ is the angular velocity of the ball, C_m is the Magnus coefficient/lift coefficient and C_d is the drag coefficient. With reference to sources, the C_m and C_d were found to be 0.29 and 0.5 respectively[1].

Since it is understood that the slowdown in the average speed of the ball was enforced so that television spectators could better track the ball, we sought to determine the average tracking speed of human eyes when watching televised table tennis. This tracking speed would give us a benchmark for the minimum speed of a ping pong ball, as anything slower would inevitably lead to a less exciting game as it would be slower than the average moving thing that humans notice. The increase in average rally count in this case, is not a good indicator of fun for spectators, since it will greatly sacrifice the speed at which the game is played.

To find the average tracking speed of the human eye on a televised game of table tennis, the average angular movement of the eyes must be involved. It is found to be 30°/s [2], and if the audience sits at approximately 2m [3] away from their television, their eyes can cover a distance of 30/360 × 2π × 2m=1.047m in 1 sec.

Therefore, the minimum velocity of the ball on screen should be 1.0471m/s for optimal fun. To find the minimum velocity of the ball at the competition grounds, the velocity found above must be scaled up according to the length ratio between items on screen and items at the competition venue. A table tennis game is assumed to typically be televised on 4:3 home televisions of average 32" [4] across. Referring to an ITTF sample of optimum framing[5] as shown in Fig. 2 below, the diagonal of the table apparently takes up 34% of the diagonal of the screen. Some calculations reveal that the screen image is scaled down 11.35 times as compared to the live event.

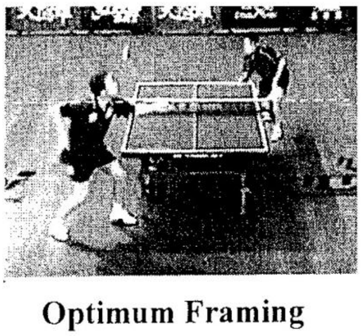


Fig. 2 Optimum framing of a Table Tennis match

Therefore, the minimum or optimal velocity of the ball on the competition grounds should be V_{op} = 11.35 × 1.0471m/s = 11.88m/s for optimal fun.

ANALYSIS

To determine the best size of table tennis ball, we have concluded that the ball should propagate as close as possible to the optimal speed of 11.88m/s as it crosses the net. We chose the net as a good reference point since spectators may not be able to properly track a ball at impact with a paddle. Midway across the table is a reasonable point for reference and it also gives the ball enough time and space to decelerate to its stable velocity.

Our analysis is conducted for balls of varying diameters, not more than 44mm [6] since balls any larger are currently not in production for common usage. Furthermore, our analysis in the later part of this report also reinforced this reason to not consider balls larger than 44mm.

A. Direct Smash

For our first scenario of a direct smash, the ball is assumed to have negligible spin in any direction or axis. It is also assumed to possess a large initial horizontal velocity and a relatively large downward velocity. The diagram below provides hypothesises an illustration of the smash, not drawn to scale.

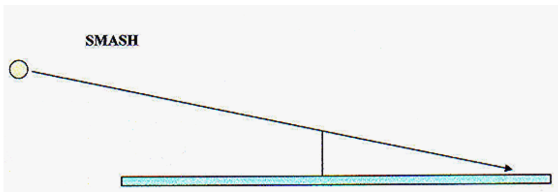


Fig 3. Direct smash with no spin

The starting horizontal velocity V_x is set to be 25m/s[7] while the vertical velocity V_z is set at -5m/s where up is positive. This value of horizontal velocity is chosen as it matches the top horizontal speeds of table tennis balls from our source. With $\omega_0 = 0$, The following corrected equation of motion is derived and used for this scenario:

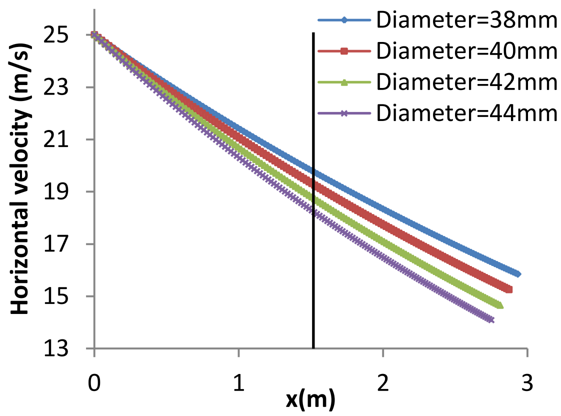
$$m_{ball}\frac{d^2}{dt^2}\vec{r} = m_{ball}\vec{g} - \frac{C_d\rho A}{2}|\vec{v}|\vec{v} \qquad (2)$$

Let (x,y,z) be the local coordinates of the ball(the reason will be shown later) and assuming no side-curve shots:

$$m_{ball}\frac{d}{dt}\begin{pmatrix} v_x \\ 0 \\ v_z \end{pmatrix} = m_{ball}\begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix} - \frac{C_d\,\rho\,\pi\,r_{ball}^2}{2}\sqrt{v_x^2 + v_y^2}\begin{pmatrix} v_x \\ 0 \\ v_z \end{pmatrix} \qquad (3)$$

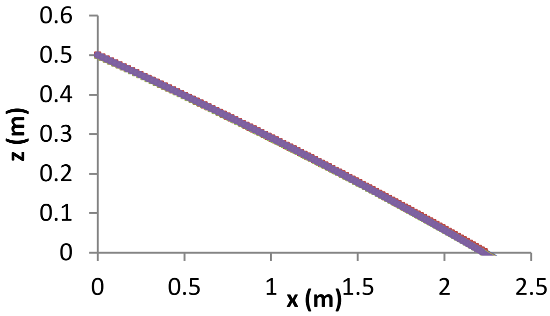
As observed, Equation 3 is composed of two coupled non-linear differential equations. Numerical analysis, using the Runge-Kutta method, is then used to find the variation of the velocity with time for a certain value of diameter of the ball. In this paper, C++ (Appendix A) is the main instrument in extracting graphs and data from. In addition, a brief Scilab coding is attached for cross reference.

With the necessary assumptions, graphs of horizontal velocity versus horizontal displacement are plotted. They are plotted for several diameters of ping pong balls. The velocity of the balls at the position over the net is determined.



Graph 1. Graph of horizontal velocity vs. horizontal displacement for direct smash with x=1.56m line

As observed, the 44m ball comes closest to the optimal speed of 11.88m/s when just above the net. In the case of a smash, a larger ball will obviously lower average speed of the ball due to drag. The 1.56m line is drawn to show the point at which the ball traverses the net.



Graph 2. Graph of vertical displacement vs. horizontal displacement for direct smash

From Graph 2, it is evident that the size of the ball does not affect the trajectory of smashed balls much, since their paths overlap quite completely.

B. Top Spin

In this scenario, the ball is given a top spin with a forward loop stroke, which will yield a large Magnus effect on the ball during motion. The ball is assumed to have a large starting horizontal velocity V_x of 20m/s[8] while the vertical velocity V_z will be +1.2m/s where up is positive. These values fall in line with the average top spin ball speeds as found in our sources. The +1.2m/s upwards is necessary for the ball to traverse over the net.

Due to the low density and low mass of the ping pong ball, the Magnus force must be included as it will contribute large observable changes in the movement of the ball. The Magnus force arises due to the spin of the ball which gives rise to the differences in pressure at the ends of the ball.

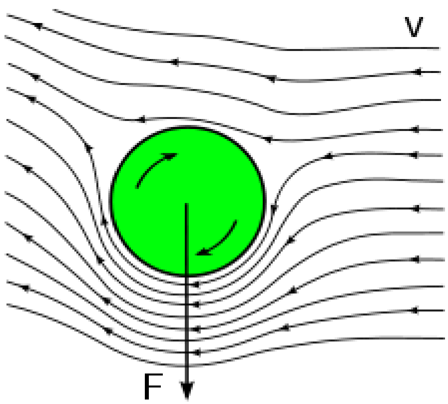


Fig 4. Magnus force F on a rotating body in a fluid with velocity v

Assuming that the angular velocity stays constant throughout the movement across the table and the direction of angular velocity is only in the local y-direction or perpendicular to both the table surface and gravity (i.e. no side spin), the equations of motions for a topspin ball can then be described below:

$$m_{ball}\frac{d}{dt}\begin{pmatrix}v_x\\0\\v_z\end{pmatrix} = m_{ball}\begin{pmatrix}0\\0\\-g\end{pmatrix} + 4\pi C_M {r_{ball}}^3\rho\,\omega_0\begin{pmatrix}v_z\\0\\-v_x\end{pmatrix} - \frac{C_d\,\rho\,\pi\,{r_{ball}}^2}{2}\sqrt{{v_x}^2+{v_y}^2}\begin{pmatrix}v_x\\0\\v_z\end{pmatrix} \qquad (4)$$

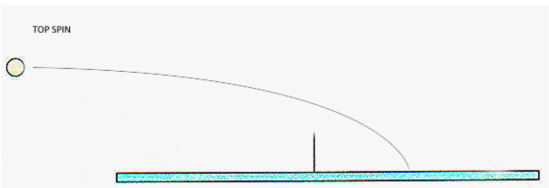


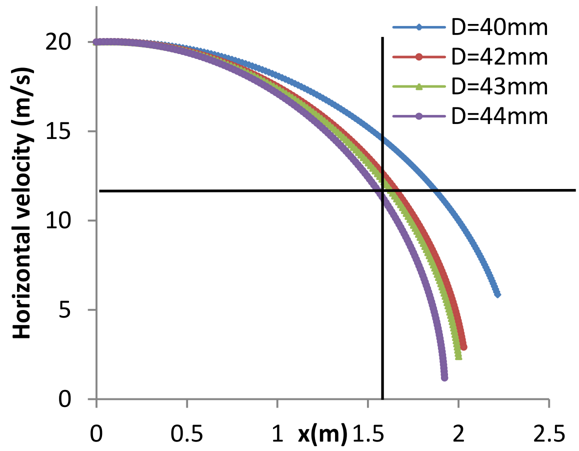
Fig 5. Loop stroke with top spin

The trajectory of a top spin ball is therefore hypothesised to appear as shown in Fig. 5. The value of ω_0 is found to be 100 revolutions/s[9], for a ball diameter of 40mm. Subsequent ω for larger balls are determined by using equation (6) as shown:

$$m_p R v_p = I_{sphere}\omega = \frac{2}{3}m_{sphere}R^2\omega \qquad (5)$$

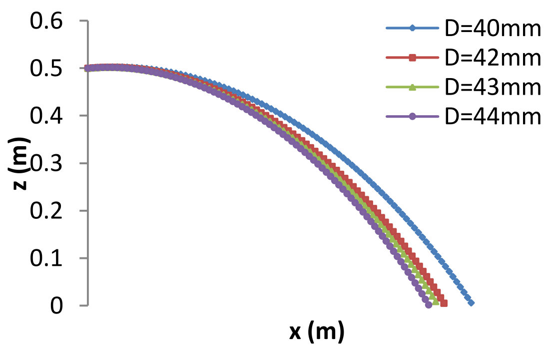
$$\omega \propto \frac{1}{R} \qquad (6)$$

The graphs of horizontal velocity versus horizontal displacement are plotted for several diameters of ping pong balls. The velocity of the balls at the position over the net is determined for comparison with the optimal velocity.



Graph 3. Graph of horizontal velocity vs. horizontal displacement for top spin with V_x=11.88m/s and x=1.56m lines

As observed, with increasing diameter, the velocity will decrease, primarily due to the drag force slowing the ball down. However, the reduced angular velocity will also mean that the horizontal velocity is not channelled into downward velocity by the Magnus force as quickly. Overall, it is observed that the 44mm ball comes closest to the optimal speed of 11.88m/s when it is just above the net. Since the optimal speed and net distance intersection lies in between 43mm and 44mm, this further justifies why we do not need to analyse data for balls larger than 44mm.



Graph 4. Graph of vertical displacement vs. horizontal displacement for top spin

The path of the top spin balls is plotted as shown above in Graph 4. It can be inferred that the resistive effect of drag, overcomes the lesser spin and curvature of path, to yield a shorter total horizontal displacement for larger balls.

C. Back Spin

In this scenario, the ball is given a back spin with a chop stroke, which will yield a large Magnus effect on the ball during motion. The ball is assumed to have starting velocities similar to that of the top spin balls. Horizontal velocity V_x is 20m/s while the vertical velocity V_z will be +1.2m/s where up is positive. The vertical velocity is once again necessary for the ball to successfully cross over the net.

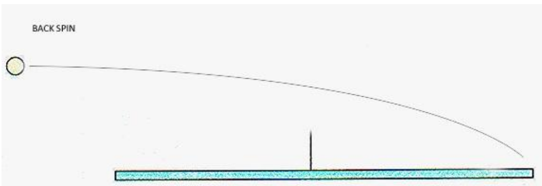
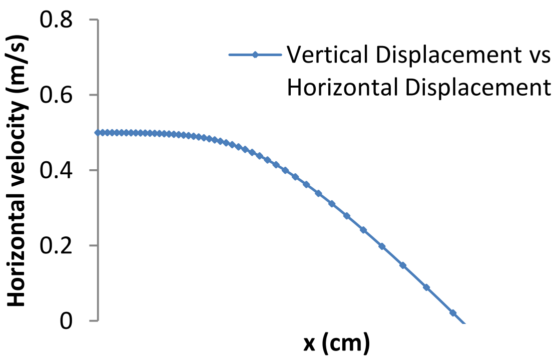


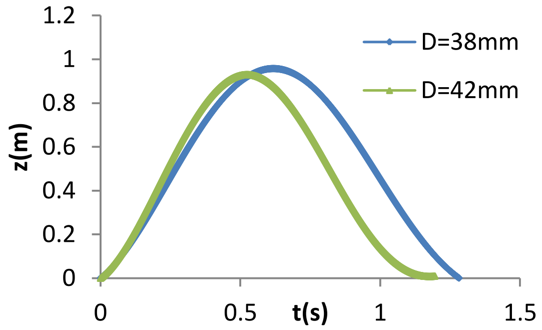
Fig 6. Chop stroke with back spin

Very much like the loop stroke for top spin, the chop stroke with back spin follows the same equation of motion as shown in Equation 4 with the same value of ω_0 of 100 rev/s. In this case, due to the nature of the chop stroke with back spin, the ball travels at a slower pace as its new upward velocity generated from the Magnus effect is negated by gravity. The analysis here is therefore only useful for corroborating the hypothesised trajectory as seen in Fig 6 above. The results from our analysis yielded the following in Graph 5.



Graph 5. Graph of horizontal velocity vs. horizontal displacement for back spin

The graph corroborates well with our expected observation due to the observably smooth decent brought about from the lifting Magnus force.



Graph 5. Graph of vertical displacement vs. time for back spin

Assuming that the initial height of the ball (z) is at the same level as the table, its initial horizontal and vertical velocity taken to be 5m/s and 1m/s respectively (These values are chosen such that the ball is able to go over the net, and hitting the far end of the table), and the average value of the backspin is 50rev/s [8], a graph of the height of the ball against time for 2 different diameters is plotted as above. It is observed for both diameters of the ball that the value of z , when it returns to the table, falls to zero over a long period of time (~1 s). This shows that the velocity of the ball is very slow as compared to the forward smash and loop. Thus, backspin motion is not taken into account in finding the optimal diameter size since most balls with different diameters will all move in a slow speed.

DISCUSSION

In the course of the investigation into the problem, the following assumptions were made:

1) The medium that the ball travel in is a still body, without any net air current. Any air current will cause the ball's trajectory to be different as calculated.

2) The ball is a perfect sphere, with a uniform weight distribution. Due to manufacturers' tolerance, there will be a slight non uniformity of the ball, as defined

as "veer"[10] in the official table tennis rules. This will shift its centre of mass and cause the effects of the spin of the ball to be different as calculated.

3) The ball with different diameters have the same mass. In the actual alterations of the official rules, the mass of the table tennis ball is increased from 2.5g to 2.7g[11]. This change in mass was ignored so as to determine the effects of the trajectory solely by the change in diameter.

4) The psychological effects of using a ball with a different diameter on the spectators were ignored. A survey[11] done in Japan, while there were general consensus that the rally length seemed to be lengthen (72.1%) and that the movements of the ball is easier to see (62.9%) for 40mm ball compared to the 38mm ball, 41.1% of the respondents felt that the games using the 40mm balls are of the same excitement levels as the 38mm ball games, close to the 48.7% who felt that the bigger balls bring better excitement. As a result, we only used the criteria of visual tracking speed to classify our findings.

5) Turbulent effects are considered to be negligible in this investigation, since the form of the Magnus Force used does not incorporate turbulent effects.

6) Vibrations of the ball from impact are considered to be negligible, due to its high coefficient of restitution and rigidity.

7) Analysed table tennis ball diameter was restricted to round integers of millimetres, since manufacturers have large margins of error or tolerance.

Regarding the strengths of our chosen analysis, our usage of a computational approach provides good comprehensive insights, by taking into account the various factors that may influence the trajectory of the ball. Our choice to use the visual acuity of television spectators as a benchmark also brings a new dimension to our analysis, especially when televised games can better penetrate the masses. Television spectators also feel more removed than live spectators, thus require additional effort to bring the fun into their homes.

A possible weakness of this analysis could stem from the fixed starting conditions used for each step of our investigation. Due to limited resources, we are excluding a whole range of other possible starting values of velocity, displacement or angular velocity.

CONCLUSION

The optimal ping pong ball size for the most satisfaction provided to television viewers was determined to be 44mm. Overall, the analysis provided accurate predictions of velocity and displacement of different sized table tennis balls when subjected to a direct smash, loop or chop stroke.

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Appendix

Mass of ball(g)	*2.3g*
Drag Coefficient	*0.5*
Magnus Coefficient	*0.29*
Density of air(kg/m³)	*1.225*
Table dimensions	*As per Figure 1*

Table of input factors

Coding for Scilab

```
-->deff('[ff]=f(t,x)','ff=[-b*x(2)-k*x(1)*sqrt(x(1)^2+x(2)^2);(b*x(1)-
k*x(2)*sqrt(x(1)^2+x(2)^2))-g]')

-->k=0.151;g=9.81;b=30;t0=0;tn=0.1;Dt=0.001;t=[t0:Dt:tn];x0=[25;-5];
//k=(r^2)*pi*Cd*(p/2m),b=0.29*4πp(ω/m)*r^3

-->x=ode(x0,t0,t,f);

-->xtitle('Velocity vs Time-No spin','t(s)','v(m/s)')

-->plot2d([t',t'],[x(1,:)',x(2,:)'],[1,9],'111','x1@x2',[0 -10 0.1 35])
```

Code for C++: Runge-kutta method

```
#include<stdio.h>
#include<math.h>
int main()
{
double t,z,x,vx,vz,xmax,dt,k1,k2,k3,k4,l1,l2,l3,l4;
double c1,c2;
double f(double t, double vx,double vz);
double g(double t, double vx,double vz);
int i; //specify the range//
//Specify conditions//
	xmax=50.0;
	dt=0.001;
//intitial conditions//
	vz=-5.0;
	vx=25.0;
	t=0;
	x=0;
	z=0.5;
for( i=0;i<200;i++)
{	printf("%lf\n",vx);//print type of variables chosen
			k1=dt*f(t,vx,vz);
			l1=dt*g(t,vx,vz);
			k2=dt*f(t+dt/2,vx+k1/2,vz+l1/2);
			l2=dt*g(t+dt/2,vx+k1/2,vz+l1/2);
			k3=dt*f(t+0.5*dt,vx+k2/2,vz+l2/2);
			l3=dt*g(t+0.5*dt,vx+k2/2,vz+k2/2);
			k4=dt*f(t+dt,vx+k3,vz+l3);
			l4=dt*g(t+dt,vx+k3,vz+l3);
			vx+=(k1+2*k2+2*k3+k4)/6;
			vz+=(l1+2*l2+2*l3+l4)/6;
			x+=dt*(vx+(k1+k2+k3)/6);
			z+=dt*(vz+(l1+l2+l3)/6);
	t+=dt;
}
return 0;
```

```
}

double f(double t,double vx,double vz)
{
	//insert the differential eqn here with the corresponding parameters:eg r=0.04, 0.29*4πp(ω/m)=567832, pi*Cd*(p/2m)=418.3//

		return -567832*(0.04*0.04*0.04)*vz-(0.04*0.04)*418.3*vx*sqrt((vx)*(vx)+(vz)*(vz));
}

double g(double t,double vx,double vz)
{
	//insert the differential eqn here with the corresponding coefficient of magnus force//

	return  567832*(0.04*0.04*0.04)*vx-  (0.04*0.04)*418.3*vz*sqrt((vx)*(vx)+(vz)*(vz))-9.81;
}
```